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J. D. Esary

A. W. Marshall

F. Proschan

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DETERMINING AN APPROXIMATE CONSTANT FAILURE RATE FOR A SYSTEM WHOSE COMPONENTS HAVE CONSTANT FAILURE RATES

by

J. D. Esary

A. W. Marshall

and

F. Proschan

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This manuscript was prepared for, and presented to, the NATO Reliability Conference, Turin, 1969. A slightly more general version of some of the results described here can be found in Boeing document D1-82-0805 (Dec. 1968), "Reliability Applications of the Hazard Transform" by the same authors. Other results from D1-82-0805 have been separately prepared for publication (see the fourth reference to this manuscript).

Abstract

It is often necessary to predict the reliability of a system when only the mean lifetimes of its components are known. Then it is usually assumed that each component has a constant failure rate (equal to the reciprocal of its mean lifetime) and that component failures occur independently. It is customary to express the prediction in terms of some constant failure rate for the system. Questions arise as to when this procedure is precise, and if it is not precise, what constant values for the system failure rate give reasonable approximations for the actual system failure rate function.

In this paper these questions are answered in the case that components (a) fail independently and (b) have constant failure rates. Then it is well known that the failure rate of a series system is constant and equal to the sum of the component failure rates. *When* (a) *and* (b) *hold, the system failure rate can be constant only for a series system. For other than series systems approximate constant failure rates should be chosen to lie between two bounds that can be computed from the component failure rates.* (The lower bound is the sum of the component failure rates for those components that can cause system failure by their single, isolated failure. The upper bound is the smallest sum of component failure rates that can be obtained for any set of components that can insure the functioning of the system by all functioning.) *For such choices the predicted system reliability will be accurate for a mission of some particular duration, smaller than the actual system reliability for shorter missions, and larger than the actual system reliability for longer missions.* For any other choice the prediction will always be too large or too small.

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1. Introduction

It is a familiar practice to convert a given, or estimated, mean lifetime (mean time to failure) for a device into a reliability prediction by assuming that its lifetime has an exponential distribution, i.e.,

(1.1) *If* $T \geq 0$ *is the random lifetime of a device, and* $m = E(T)$ *is its mean lifetime, then its reliability* $P[T>t]$ *for a mission of duration* t *is approximated by*

$$P[T>t] = e^{-\lambda t}, \quad t \geq 0,$$

where $\lambda = 1/m$.

The procedure described in (1.1) is equivalent to assuming that the device has a constant hazard (failure) rate $\lambda = 1/m$.

When the mean lifetimes of components are used to predict the reliability of the system, it is also customary to express the prediction in terms of a mean lifetime for the system with the understanding that (1.1) will be used to approximate reliabilities. It is also usual in arriving at the prediction to assume that components fail independently.

The practices mentioned above raise the following questions: *Assuming that the lifetimes of components are statistically independent and have exponential distributions, then:*

(1.2) *For what systems does the system lifetime actually* *have an exponential distribution?*

(1.3) *For a system whose lifetime is not exponentially distributed, which exponential distributions give reasonable approximations to the actual system lifetime distribution?*

Part of the answer to Question (1.2) is very well known. If the lifetimes of the components in a series system (i.e., a system that fails with its first component failure) have exponential distributions, then the system lifetime has an exponential distribution. We show that only series systems have this property.

It is also known that if the lifetimes of the components in a coherent system have exponential distributions, then the system lifetime has an increasing hazard (failure) rate average (IHRA) distribution [Birnbaum, Esary, and Marshall, 1966]. We answer Question (1.3) by exhibiting an interval in which an approximate, constant system failure rate must lie if the corresponding approximate, exponential distribution for the system lifetime is to give the same reliability prediction as the actual IHRA distribution, at least for a mission of some duration t.

Notation. We use the standard vector notation $\underset{\sim}{a} = (a_1,\ldots,a_n)$, $c\underset{\sim}{a} = \underset{\sim}{a}c = (ca_1,\ldots,ca_n)$, and the special convention $e^{-\underset{\sim}{a}} = (e^{-a_1},\ldots,e^{-a_n})$.

2. The Hazard Transform of a Coherent System

The *structure function* $\phi(x_1,\ldots,x_n)$ describes the organization of a system; x_i indicates the state of the i^{th} component, with $x_i = 1$ if the component is functioning and $x_i = 0$ if the component is failed, and ϕ indicates the corresponding state of the system, with $\phi(\underset{\sim}{x}) = 1$ if the system is functioning and $\phi(\underset{\sim}{x}) = 0$ if the system is failed. A system is *coherent* if its structure function is increasing in each of its arguments and is not constant in any of its arguments.

The *reliability function* $h(p_1,\ldots,p_n)$ is defined by

$$h(\underset{\sim}{p}) = P[\phi(X_1,\ldots,X_n) = 1], \quad 0 \leq p_i \leq 1, \; i = 1,\ldots,n, \tag{2.1}$$

where $X_1,\ldots,X_n$ are independent Bernoulli random variables with $P[X_i=1] = p_i$, and ϕ is the structure function of the system.

The *hazard transform* $\eta(\rho_1,\ldots,\rho_n)$ is defined by

$$\eta(\underset{\sim}{\rho}) = -\log h(e^{-\rho_1},\ldots,e^{-\rho_n}), \quad \rho_i \geq 0, \; i = 1,\ldots,n, \tag{2.2}$$

where h is the reliability function of the system.

In this paper a special role is played by the *series* system for which $\phi(\underset{\sim}{x}) = \prod_{i=1}^{n} x_i$, $h(\underset{\sim}{p}) = \prod_{i=1}^{n} p_i$, and $\eta(\underset{\sim}{\rho}) = \sum_{i=1}^{n} \rho_i$.

In the context of Questions (1.2) and (1.3) the components in a coherent system are assumed to have independent random lifetimes $T_1,\ldots,T_n$. Let T be the corresponding random lifetime for the system. Then it follows from (2.1) that

$$P[T>t] = h\{P[T_1>t], \ldots, P[T_n>t]\}, \quad t \geq 0, \tag{2.3}$$

by choosing the Bernoulli random variables so that $X_i = 1 \iff T_i > t$, $i = 1,\dots,n$, and noting that $T > t \iff \phi(\underset{\sim}{X}) = 1$. If $R(t) = -\log P[T>t]$, $t \geq 0$, i.e., $P[T>t] = e^{-R(t)}$, and $R_i(t) = -\log P[T_i>t]$, $t \geq 0$, $i = 1,\dots,n$, then it follows from (2.3) and (2.2) that

$$R(t) = \eta\{R_1(t),\dots,R_n(t)\}, \quad t \geq 0. \tag{2.4}$$

R is the *hazard function* of the random lifetime T and $R_1,\dots,R_n$ are the hazard functions of $T_1,\dots,T_n$. *The hazard transform of a coherent system expresses the hazard function of the system lifetime in terms of the hazard functions of the component lifetimes.*

For our purposes the hazard transform is a useful description of a coherent system. If the component lifetimes have exponential distributions, i.e., $P[T_i>t] = e^{-\lambda_i t}$, $t \geq 0$, then their hazard functions are $R_i(t) = \lambda_i t$, $i = 1,\dots,n$. Then (2.4) becomes

$$R(t) = \eta(\lambda_1 t,\dots,\lambda_n t), \quad t \geq 0, \tag{2.5}$$

i.e., questions such as (1.2) and (1.3) reduce to studies of the behavior of the hazard transform on rays $\{\underset{\sim}{\lambda} t, t\geq 0\}$ in its domain.

In what follows we will need certain properties of the hazard transform of a coherent system:

(2.6) $\eta(0,\dots,0) = 0$, *and* $\eta(\rho_1,\dots,\rho_n) > 0$ *whenever* $\rho_i > 0$, $i = 1,\dots,n$.

(2.7) η *is increasing in each of its arguments.*

(2.8) η *is starshaped, i.e.,* $\eta(a\underset{\sim}{\rho}) \leq a\eta(\underset{\sim}{\rho})$ *for all* $\underset{\sim}{\rho}$ *and all* $0 \leq a \leq 1$.

(2.9) *If* $\eta(a\underset{\sim}{\rho}) = a\eta(\underset{\sim}{\rho})$ *for some* $\underset{\sim}{\rho}$ *such that* $\rho_i > 0$, $i = 1,\ldots,n$, *and some* $0 < a < 1$, *then the system is a series system.*

The first two properties above are equivalent to familiar elementary properties of the structure and reliability functions of coherent systems. Property (2.8) comes from an inequality for the reliability function obtained by Birnbaum, Esary, and Marshall (1966, Theorem 2.1). The details of the translations of properties (2.6) through (2.8) into hazard terminology and the proof of property (2.9) are given in Esary, Marshall, and Proschan (1969).

Other examples of coherent systems are given in Birnbaum, Esary, and Saunders (1961) and Barlow and Proschan (1965).

3. Exponential System Lifetimes

If the components in a series system have independent, exponentially distributed lifetimes, then it is easy to confirm the well-known fact that the system lifetime is exponentially distributed. From (2.5) and the definition of a series system

$$R(t) = \sum_{i=1}^{n} \lambda_i t = \left(\sum_{i=1}^{n}\lambda_i\right)t, \quad t \geq 0, \tag{3.1}$$

where R is the system hazard function and $\lambda_1,\ldots,\lambda_n$ are the component hazard rates, i.e., R is the hazard function of an exponential distribution.

For the remainder of this paper we will explicitly assume *that the hazard rate of an exponentially distributed lifetime is not zero.* With this assumption the ray $\{\underset{\sim}{\lambda}t, t\geq 0\}$ determined by the hazard functions of a set of exponentially distributed component lifetimes is an *interior ray*, i.e., $\lambda_i > 0$, $i = 1,\ldots,n$. (We have already tacitly assumed that exponential hazard rates are finite, e.g., in writing (2.5) when the hazard transform is defined for finite arguments.)

The rest of the answer to question (1.2) is then contained in the following proposition:

(3.2) Proposition. *Suppose that the lifetimes of the components in a coherent system are independent and exponentially distributed. If the system lifetime is exponentially distributed, then the system is a series system.*

Proof. If the system lifetime is exponentially distributed, then $R(t) = \mu t$, $t \geq 0$, for some $\mu > 0$, where R is the hazard function of the system lifetime. Then (2.5) becomes $\eta\{\underset{\sim}{\lambda} t\} = \mu t$, $t \geq 0$, where η is the hazard transform of the system and $\lambda_i > 0$, $i = 1,\ldots,n$, are the component hazard rates. Consider some $t > 0$ and some $0 < a < 1$. Let $\rho_i = \lambda_i t$, $i = 1,\ldots,n$. Then $\eta(a\underset{\sim}{\rho}) = \eta(a\underset{\sim}{\lambda} t) = \eta\{\underset{\sim}{\lambda}(at)\} = \mu(at) = a(\mu t) = a\eta(\underset{\sim}{\lambda} t) = a\eta(\underset{\sim}{\rho})$. Since $\rho_i > 0$, $i = 1,\ldots,n$, the system must be a series system by property (2.9). $\square$

4. Initial and Terminal System Hazard Rates

When the components in a coherent system have independent, exponentially distributed lifetimes, it follows from (2.5) that the *hazard rate* at time t, $R'(t) = dR(t)/dt$, for the system lifetime is given by

(4.1) $$R'(t) = \eta'(\lambda_1 t,\ldots,\lambda_n t), \quad t \geq 0,$$

where $\eta'(\underset{\sim}{\lambda}t) = d\eta(\underset{\sim}{\lambda}t)/dt$, η is the hazard transform of the system, R is the system hazard function, and $\lambda_1,\ldots,\lambda_n$ are the component hazard rates. For a non-series system, R is not the hazard function of an exponential distribution by Proposition 3.2, so that R' is not a constant function of t. In Section 5 it is shown that any constant approximation to R' should lie between $R'(0)$ and $\lim_{t\to\infty}R'(t)$. This section is devoted to deriving convenient expressions for these bounds.

The i^{th} component in a coherent system is a *cut* of the system if its failure is sufficient to cause the failure of the system, i.e., if $x_i = 0$ implies $\phi(\underset{\sim}{x}) = 0$ where ϕ is the structure function of the system. The derivation which follows shows that on a ray $\{\underset{\sim}{\lambda}t, t\geq 0\}$

(4.2) $$\eta'(\underset{\sim}{\lambda}0) = \sum_{i\varepsilon K} \lambda_i,$$

where K is the set of all components that are cuts of the system. We say that $\mu_0 = \sum_{i\varepsilon K} \lambda_i$ is the *initial hazard rate* for the system on the ray since from (4.1) $R'(0) = \mu_0$.

A set of components P in a coherent system is a *path set* if the functioning of each component in P insures the functioning of the system, i.e., if $x_i = 1$ for all $i \varepsilon P$ implies $\phi(\underset{\sim}{x}) = 1$. The derivation which follows shows that on an interior ray $\{\underset{\sim}{\lambda}t, t\geq 0\}$

(4.3) $$\lim_{t\to\infty}\eta'(\underset{\sim}{\lambda}t) = \mu_\infty,$$

where

$$\mu_\infty = \min\{\textstyle\sum_{i\varepsilon P}\lambda_i;\ P \text{ is a path set}\}.$$

We say that μ_∞ is the *terminal hazard rate* for the system on the ray since from (4.1) $\lim_{t\to\infty} R'(t) = \mu_\infty$. The same derivation shows that

(4.4) $$\lim_{t\to\infty}\{\mu_\infty t - \eta(\underset{\sim}{\lambda} t)\} = \log r,$$

where r is the number of path sets P for which $\sum_{i\varepsilon P}\lambda_i = \mu_\infty$.

For a series system, $\mu_0 = \mu_\infty = \sum_{i=1}^n \lambda_i$ from (3.1). The same conclusion can be obtained directly from the definitions of μ_0 and μ_∞ since on the one hand each component in a series system is a cut of the system and on the other hand there is only one path set for a series system, i.e., the set of all components. For a non-series system $K \subset P$ for any path set P, and furthermore $K \neq P$. Thus *for a non-series system and an interior ray*, $\mu_0 < \mu_\infty$.

The remainder of this section consists of a derivation to confirm (4.2), (4.3), and (4.4).

From (2.3)

(4.5) $$\eta'(\underset{\sim}{\lambda} t) = \frac{-h'(e^{-\underset{\sim}{\lambda} t})}{h(e^{-\underset{\sim}{\lambda} t})}, \quad t \geq 0,$$

where $h'(e^{-\underset{\sim}{\lambda} t}) = dh(e^{-\underset{\sim}{\lambda} t})/dt$ and h is the reliability function of the system.

From (2.1), $h(\underset{\sim}{p}) = E\{\phi(\underset{\sim}{X})\}$, so that

(4.6) $$h(e^{-\underset{\sim}{\lambda} t}) = \sum_{\underset{\sim}{x}} \phi(\underset{\sim}{x})P(\underset{\sim}{x},\underset{\sim}{\lambda},t), \quad t \geq 0,$$

where ϕ is the structure function of the system,

$$(4.7) \qquad P(\underset{\sim}{x},\underset{\sim}{\lambda},t) = \prod_{i\varepsilon C_1(\underset{\sim}{x})} e^{-\lambda_i t} \prod_{i\varepsilon C_0(\underset{\sim}{x})} (1-e^{-\lambda_i t}),$$

and $C_1(\underset{\sim}{x}) = \{i:x_i=1\}$, $C_0(\underset{\sim}{x}) = \{i:x_i=0\}$. From (4.6)

$$(4.8) \qquad h'(e^{-\underset{\sim}{\lambda}t}) = \sum_{\underset{\sim}{x}} \phi(\underset{\sim}{x}) P'(\underset{\sim}{x},\underset{\sim}{\lambda},t), \quad t \geq 0,$$

where $P'(\underset{\sim}{x},\underset{\sim}{\lambda},t) = dP(\underset{\sim}{x},\underset{\sim}{\lambda},t)/dt$. From (4.7)

$$(4.9) \qquad P'(\underset{\sim}{x},\underset{\sim}{\lambda},t) = -\left(\sum_{i\varepsilon C_1(\underset{\sim}{x})} \lambda_i\right) P(\underset{\sim}{x},\underset{\sim}{\lambda},t)$$

$$+ \sum_{i\varepsilon C_0(\underset{\sim}{x})} \lambda_i P\{(1_i,\underset{\sim}{x})\underset{\sim}{\lambda},t\},$$

where $(1_i,\underset{\sim}{x}) = (x_1,\ldots,x_{i-1},1,x_{i+1},\ldots,x_n)$.

From (4.7)

$$(4.10) \qquad P(\underset{\sim}{x},\underset{\sim}{\lambda},0) = \begin{matrix} 1 & \text{if } \underset{\sim}{x} = \underset{\sim}{1} \\ 0 & \text{otherwise,} \end{matrix}$$

where $\underset{\sim}{1} = (1,\ldots,1)$. Then from (4.9) and (4.10)

$$(4.11) \qquad P'(\underset{\sim}{x},\underset{\sim}{\lambda},0) = \begin{matrix} \sum_{i=1}^{n} \lambda_i & \text{if } \underset{\sim}{x} = \underset{\sim}{1} \\ \lambda_i & \text{if } \underset{\sim}{x} = (0_i,\underset{\sim}{1}), \quad i = 1,\ldots,n, \\ 0 & \text{otherwise,} \end{matrix}$$

where $(0_i,\underset{\sim}{x}) = (x_1,\ldots,x_{i-1},0,x_{i+1},\ldots,x_n)$. Then from (4.6) and (4.10)

$$(4.12) \qquad h(e^{-\underset{\sim}{\lambda}0}) = \phi(\underset{\sim}{1}) = 1,$$

and from (4.8) and (4.11)

$$(4.13)\qquad h'(e^{-\lambda 0}) = -\phi(\underset{\sim}{1}) \textstyle\sum_{i=1}^{n}\lambda_i + \sum_{i=1}^{n}\phi(0_i,\underset{\sim}{1})\lambda_i$$

$$= -\textstyle\sum_{i\in K}\lambda_i .$$

Then (4.2) follows from (4.5), (4.12), and (4.13).

Let $\mu(\underset{\sim}{x},\underset{\sim}{\lambda}) = \sum_{i\in C_1(\underset{\sim}{x})}\lambda_i$. From (4.7) with $\lambda_i > 0$, $i = 1,\ldots,n$,

$$(4.14)\qquad \lim_{t\to\infty} \frac{P(\underset{\sim}{x},\underset{\sim}{\lambda},t)}{e^{-\mu t}} = \begin{cases} 1 & \text{if } \mu = \mu(\underset{\sim}{x},\underset{\sim}{\lambda}) \\ 0 & \text{if } \mu < \mu(\underset{\sim}{x},\underset{\sim}{\lambda}). \end{cases}$$

Then from (4.9) and (4.14)

$$(4.15)\qquad \lim_{t\to\infty} \frac{P'(\underset{\sim}{x},\underset{\sim}{\lambda},t)}{e^{-\mu t}} = \begin{cases} -\mu & \text{if } \mu = \mu(\underset{\sim}{x},\underset{\sim}{\lambda}) \\ 0 & \text{if } \mu < \mu(\underset{\sim}{x},\underset{\sim}{\lambda}). \end{cases}$$

Then from (4.6) and (4.14)

$$(4.16)\qquad \lim_{t\to\infty} \frac{h(e^{-\underset{\sim}{\lambda} t})}{e^{-\mu_\infty t}} = \sum_{\underset{\sim}{x}}\phi(\underset{\sim}{x}) \lim_{t\to\infty} \frac{P(\underset{\sim}{x},\underset{\sim}{\lambda},t)}{e^{-\mu_\infty t}}$$

$$= \sum_{\{\underset{\sim}{x}:\phi(\underset{\sim}{x})=1,\mu(\underset{\sim}{x},\underset{\sim}{\lambda})=\mu_\infty\}} 1 = r.$$

Similarly from (4.8) and (4.15)

$$(4.17)\qquad \lim_{t\to\infty} \frac{h'(e^{-\underset{\sim}{\lambda} t})}{e^{-\mu_\infty t}} = -r\mu_\infty .$$

Then (4.3) follows from (4.5), (4.16), and (4.17), since

$$\lim_{t\to\infty} \eta'(\underset{\sim}{\lambda} t) = \lim_{t\to\infty} \frac{-h'(e^{-\underset{\sim}{\lambda} t})/e^{-\mu_\infty t}}{h(e^{-\underset{\sim}{\lambda} t})/e^{-\mu_\infty t}}$$

$$= \frac{r\mu_\infty}{r} = \mu_\infty .$$

From (2.2)

$$\eta(\underset{\sim}{\lambda}t) - \mu_\infty t = -\log \frac{h(e^{-\underset{\sim}{\lambda}t})}{e^{-\mu_\infty t}} ,$$

so that (4.4) follows from (4.16).

5. Exponential Approximations for System Lifetimes

If the components in a coherent system have independent, exponentially distributed lifetimes, then from (3.1) and Proposition (3.2) the system lifetime is exponentially distributed if and only if the system is a series system. Question (1.3) asks what exponential distributions are reasonable approximations to the actual distribution of the system lifetime when the system is not a series system. The answer to Question (1.3) is contained in the following proposition:

5.1 Proposition. *Let* R *be the hazard function of the lifetime of a coherent system that is not a series system. Suppose that the component lifetimes are independent and exponentially distributed with hazard rates* $\lambda_i > 0$, $i = 1,\ldots,n$. *Let* μ_0 *be the initial hazard rate for the system on the ray* $\{\underset{\sim}{\lambda}t, t\geq 0\}$ *and* μ_∞ *be the terminal hazard rate on the ray. Then:*

(i) *If* $\mu_0 < \mu < \mu_\infty$, *there exists a* $t_\mu > 0$ *such that* $R(t_\mu) = \mu t_\mu$. *Also* $R(t) < \mu t$ *if* $0 < t < t_\mu$ *and* $R(t) > \mu t$ *if* $t > t_\mu$.

(ii) $\mu_0 t < R(t) < \mu_\infty t$ *for all* $t > 0$.

Proof. From (2.5), $R(t) = \eta(\underset{\sim}{\lambda}t)$, $t \geq 0$, where η is the hazard transform of the system. Since the system is not a series system, $0 \leq \mu_0 < \mu_\infty$ [cf. Section 4].

(i) From (4.2) $\eta'(\underset{\sim}{\lambda}0) = \mu_0$ and from (4.4) $\eta(\underset{\sim}{\lambda}t)$ is asymptotic to the straight line $\mu_\infty t - \log r$ as $t \to \infty$. Thus if $\mu_0 < \mu < \mu_\infty$, then there is a $t_\mu > 0$ such that $\eta(\underset{\sim}{\lambda}t_\mu) = \mu t_\mu$. Since the system is not a series system and $\lambda_i > 0$, $i = 1,\ldots,n$, it follows from properties (2.8) and (2.9) that η is strictly starshaped on the ray $\{\lambda t, t \geq 0\}$, i.e., $\eta\{\underset{\sim}{\lambda}(at)\} < a\eta(\underset{\sim}{\lambda}t)$ for all $t > 0$ and all $0 < a < 1$. Suppose $0 < t < t_\mu$ and let $a = t/t_\mu$. Then $0 < a < 1$ and $\eta(\underset{\sim}{\lambda}t) = \eta\{\underset{\sim}{\lambda}(at_\mu)\} < a\eta(\underset{\sim}{\lambda}t_\mu) = a\mu t_\mu = \mu t$. Similarly if $t > t_\mu$, then $\eta(\underset{\sim}{\lambda}t) > \mu t$.

(ii) This conclusion can be proved by arguments similar to those used for part (i), but is also immediate from the definitions of μ_0 and μ_∞. $\square$

Proposition (5.1) is illustrated for two simple systems in Figure 1.

Recall that the hazard function R of the system lifetime T is related to the reliability of the system for a mission of duration t by $P[T>t] = e^{-R(t)}$, and that using an exponential distribution for T with hazard rate $\mu > 0$ leads to the approximation $P[T>t] \approxeq e^{-\mu t}$. Part (i) of Proposition (5.1) says that *if* $\mu_0 < \mu < \mu_\infty$ *an exponential approximation gives the correct reliability for a mission of some*

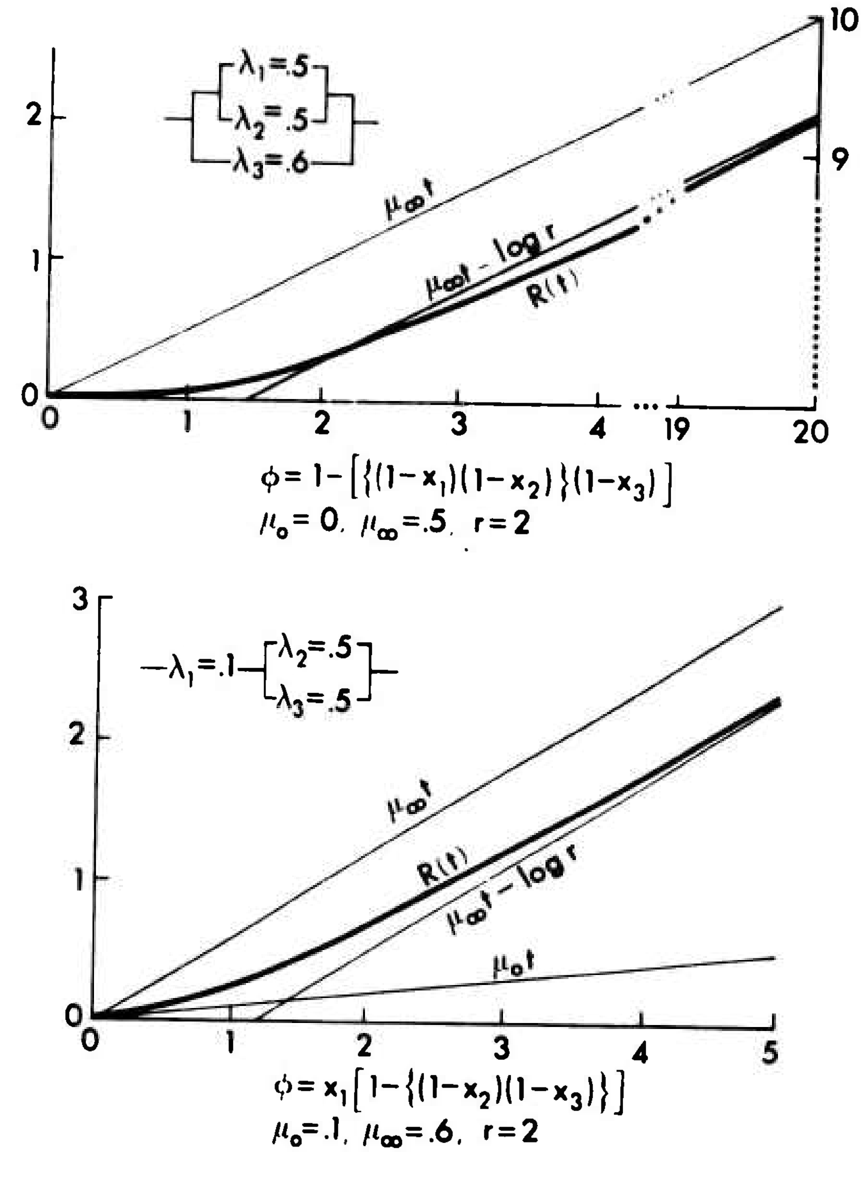

Figure 1. System hazard functions when components have constant failure rates.

duration $t_\mu > 0$, *too large a reliability for missions of duration* t *such that* $0 < t < t_\mu$, *and too small a reliability for missions of duration* $t > t_\mu$. Part (ii) says that *if* $\mu \leq \mu_0$ *an exponential approximation gives too large a reliability for all missions*, and *if* $\mu \geq \mu_\infty$ *an exponential approximation gives too small a reliability for all missions*.

If it is feasible to evaluate the reliability function h of a coherent system for a given set of arguments, then there is a simple, standard method for determining an approximate constant system hazard rate μ which gives a correct reliability prediction for a given nominal mission duration t, i.e., let

$$\mu = \frac{-\log h(e^{-\lambda_1 t},\ldots,e^{-\lambda_n t})}{t}, \tag{5.2}$$

where $\lambda_1,\ldots,\lambda_n$ are the component hazard rates. Then from (2.3) $P[T>t] = e^{-\mu t}$, where T is the system lifetime, and it follows from part (ii) of Proposition (5.1) that $\mu_0 < \mu < \mu_\infty$.

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